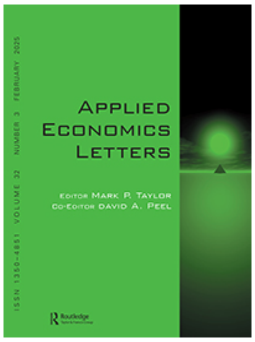

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Fractal portfolio strategies: does scale preference of investors matter?

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ABSTRACT

The mean-DCCA portfolio is known to consider the assets' nonlinearity and scaling properties by embedding the fractal correlation into the mean-variance criterion, with specific strategies under the assumption that the scale preference of investors is constant. We examine whether accounting for changes in investors' scale preference in response to market conditions improves portfolio performance. A portfolio with preference on short-scales is effective under market uncertainty, while long-scale preference strategy is effective under a steady market. Our results support the Fractal Market Hypothesis and reveal the potential effect of investor heterogeneity on portfolio risk reduction.

KEYWORDS

Portfolio selection; fractal correlation; DCCA analysis; multi-scale properties

JEL CLASSIFICATION

C32; C61; G11

I. Introduction

Modern portfolio theory, which determines the allocation of investments in financial assets, requires controlling and minimizing risk to achieve diversification. In the traditional mean-variance analysis, the investor's decision-making is characterized by expected returns and variances, and the optimal combination of assets should be identical among investors (Markowitz 1952). However, since there are different types of investors with different trading strategies and investment horizons, it is unlikely that they are homogeneous in their expectations with an agreement on specific risk measures as called for the Efficient Market Hypothesis (EMH) (Fama 1991; Kristoufek 2018). Markets are rather inefficient, complex, and likely to exhibit heterogeneous behaviours (Battiston et al. 2016; Tilfani, Ferreira, and Boukfaoui 2020), and portfolio selection based on traditional approaches may not be appropriate (Kristoufek 2018; Tilfani, Ferreira, and Boukfaoui 2019; Zhang et al. 2022). According to the alternative framework of the Fractal Market Hypothesis (FMH) proposed by Peters (1994), financial series exhibit fractal properties due to the different valuations for information flows among investment horizons, thereby justifying sudden spikes in market volatility and lack of market liquidity during crashes. Zhang et al. (2022) developed the mean-DCCA analysis by incorporating the fractal correlation characteristics of multiple assets into the mean-variance portfolio strategy, where the detrended cross-correlation analysis (DCCA) function (Podobnik and Stanley 2008) was used in place of the covariance function to substitute the risk definition.

At different scales, i.e. at different investment time horizons, the portfolio risk can be defined nonlinearly and allows investment allocations from various decision-making standards.

The mean-DCCA portfolio performs well, assuming that investors' scale preference stays constant (Zhang et al. 2022). However, their preference may shift to a different level since they change positions in response to changes in economic states. Evans (1994) and Ferson and Schadt (1996) show that time-varying risk prices play an important role in the expected returns of the stock market by using conditional factor models. Motivated by the fact that investors' preference for time horizons may vary over time and that their emphasis on short scales triggers market crashes (Peters 1994), we examine whether incorporating changes in scale preference improves portfolio performance. In particular, we construct a strategy that switches preference between short and long scales in response to maximum drawdown. For the first time within the fractal portfolio framework, the idea of utilizing

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market information is introduced. This allows us to find opportunities for positions with lower market risk. Our results indicate that short-scale preference strategy gains risk control during volatile market conditions, which supports the background hypothesis of the FMH. The study contributes to a fresh understanding of how we can benefit from investors' heterogeneous behaviour in terms of portfolio allocation.

II. Methodology and data

Methodology

In the mean-variance (MV) analysis, the portfolio is selected to minimize variance under some required expected return. The mean-DCCA (MD) analysis of Zhang et al. (2022) minimizes the DCCA (covariance) function (Podobnik and Stanley 2008) instead to consider assets' fractal correlations and multiscale property.

For a given time series $\{x_t\}_{t=1}^{N}$ and $\{y_t\}_{t=1}^{N}$, we split their cumulative sums, $X(t) = \sum_{i=1}^{t} x_i$ and $Y(t) = \sum_{i=1}^{t} y_i$, into $N_s = N/s$ non-overlapping segments of length s. The division is repeated from the other end, having $2N_s$ segments in total. For each segment, we eliminate the local trend and calculate

$$f_{XY}^2(s,v) = \frac{1}{s}\sum_{t=1}^{s}\{X_v(t) - \tilde{X}_v(t)\}\{Y_v(t) - \tilde{Y}_v(t)\}, \quad (1)$$

where $\tilde{X}_v(t)$ and $\tilde{Y}_v(t)$ denotes the degree-2 polynomial fits used to detrend the vth segment of $X(t)$ and $Y(t)$, respectively. By averaging $f_{XY}^2(s,v)$ over all segments we get the DCCA function

$$F_{XY}^2(s) = \frac{1}{2N_s}\sum_{v=1}^{2N_s} f_{XY}^2(s,v). \quad (2)$$

The function is scale-dependent and characterized by long-range power-law correlations $F^2(s) \sim s^H$, providing valuable supplemental information. [1] Different correlation levels can be uncovered under different investment horizons.

The MD portfolio of n assets is constructed using DCCA and the expected portfolio return calculated from the expected return of each asset, $\bar{r}_i$. With the constraint that risk minimization is achieved under some given return greater than r_e, the investment weight $w_i(s)$ is calculated by solving the optimization problem

$$\begin{aligned} \underset{w(s)}{\text{minimize}} \quad & \sum_{i,j=1}^{n} w_i(s)w_j(s)F_{ij}^2(s) \\ \text{subject to} \quad & \sum_{i=1}^{n} \bar{r}_i w_i(s) \geq r_e, \ \sum_{i=1}^{n} w_i(s) = 1, \\ & w_i(s) \geq 0, \ i = 1, \ldots, n. \end{aligned} \quad (3)$$

Each weight is determined to meet the optimal values under each different scale s. Since the FMH explains that investors trade from all kinds of investment horizons, the impact from multiple time scales must be reflected in the optimal weights. The weight $w_i(s)$ under a single horizon is only one component of the complex multiscale market behaviour and, therefore, not appropriate to conclude as the optimal investment weight. Additional steps are required to ensure the effectiveness of MD portfolio. Following Zhang et al. (2022), we consider a set of multiple scales $S = \{s_1 = s_{\min}, s_2, \ldots, s_{n-1}, s_n = s_{\max}\}$, where $s_{\min}$ and $s_{\max}$ are the minimum and maximum elements.

Then, the optimal investment weight w_i^{opt} is defined as the weighted average of $w_i(s)$ expressed as

$$w_i^{\text{opt}} = \sum_{s \in S} \alpha_i(s) w_i(s), \quad (4)$$

where $\sum_{s \in S} \alpha_i(s) = 1$ holds and $\alpha_i(s) \in [0,1]$ represents the investor's relative preference degree for scale s. The preference degree can be adjusted depending on how we associate $\alpha_i(s)$ with s. Given the heterogeneity of investors, we consider three types of investment strategies according to their scale preference – equal preference among time scales, more preference for shorter time scales, and more preference for longer time scales. If, for example, investors have no specific preference among different investment horizons, we set $\alpha_i(s) = \frac{1}{\#S}$, where $\#S$ denotes the total number of elements in set S.

[1] For instance, the scaling exponent H helps find traces of long- and short-memory in addition to the prevailing fractal behaviours.

Data

The fractal portfolio is applied to eight empirical daily indexes of S&P500 stock, US Treasury bond (1–3 years, 10 years), US High-yield corporate bond, and US Investment-grade corporate bond (Aaa, Aa, A, Baa) for the period of 5 January 2004 to 31 December 2021. A high-yield bond provides higher yields than Investment-grades with riskier with lower credit ratings.

III. Results and discussions

Multi-time scale property of data

Before implementing the MD analysis, we investigate whether the fractal portfolio reflects the diversification effect, if any, among the datasets at study. For each pair of daily returns, we calculate the DCCA cross-correlation coefficient of Zebende (2011) at different scales defined as $\rho_{\mathrm{DCCA}}(s) = \frac{F^2_{XY}(s)}{F_{XX}(s)F_{YY}(s)}$ (Figure 1). We confirm that coefficient levels tend to vary depending on scales. This means that there exist different diversification effects at heterogeneous scales. Therefore, the MD approach can shed light on the scale-dependent property, and there may be room for portfolio improvement.

Performance under different scaling preferences

Given that the market exhibits fractal correlations in addition to different diversification effects by scale, we construct an MD portfolio that takes into account the investors' scale preferences classified into three types: long-, short-, and balanced-scales. In particular, referring to Equation (4), we set $\alpha_i(s_j) = s_j / \sum_{s\in S} s$ for long-scale preference, and $\alpha_i(s_j) = s_{\#S+1-j} / \sum_{s\in S} s$ for short-scale preference, where s_j denotes the jth element of subset $S = \{s_1, \ldots, s_j, \ldots, s_{\#S}\}$. For the balanced-scale type, the optimal weight is calculated by averaging all the scale-dependent weights. In our analysis, we use a historical data length of 2 years and employ $S = \{5, 10, 15, \ldots, 130\}$ as the subset.

Table 1 presents the maximum drawdown performance of MV and MD portfolios based on the three strategy types, given an expected annual return of 2.5%. All three types of MD generally outperform the traditional MV. More interestingly, the higher performance of MD appears to vary by market period. For example, during the global financial crisis (2008–2009), the MD with short-term preference reduces drawdown the most, whereas after market recovery (2012–2013), the MD with long-term preference reduces the most. The results imply that understanding the heterogeneity of investors and their scale inclination plays an essential role in increasing portfolio risk control. In a relatively unstable market condition, short-scale MD minimizes portfolio risk, which is consistent with the FMH concept in that investors tend to focus more on the short term in downside situations.

Out-sample performance of the switching strategy

The out-sample performance of the MD portfolio is also discussed by backtesting. We use a 2-year rolling window (520 daily returns) estimation with quarterly rebalancing. To regard the effect of the heterogeneity of investors under different market conditions, we introduce a new strategy, that is, when rebalancing the portfolio, we switch the scale preference types among the three MD strategies. If the maximum drawdown of the balanced-scale MD over the past 2 years is worse than −1.5%, we consider the market to be in an unfavourable state and select the short-scale strategy. Otherwise, we assume the market to be out of recession or not in its downtrend and expectations for longer investment horizons have increased; thus, we select the long-scale strategy. Figure 2 shows the switch and maximum drawdown results throughout the entire period, and Figure 3 compares the drawdown with other strategies. Although in 2008 a significant drawdown is observed due to the global financial crisis, the impact on portfolio performance is the least (yellow dash lines). We also find improvement in drawdown for other periods – the effect of heterogeneous investors is constantly reflected in the allocation. In other words, MD-switching strategy for the US market achieves a more risk conservative set of allocations compared to MV and other types of MD. It not only reduces maximum drawdown but also improves Value-at-Risk (VaR) and expected shortfall (ES) while increasing portfolio returns under the same 2.5% annual required return (Table 2). Therefore, dynamically changing the scale preference of the

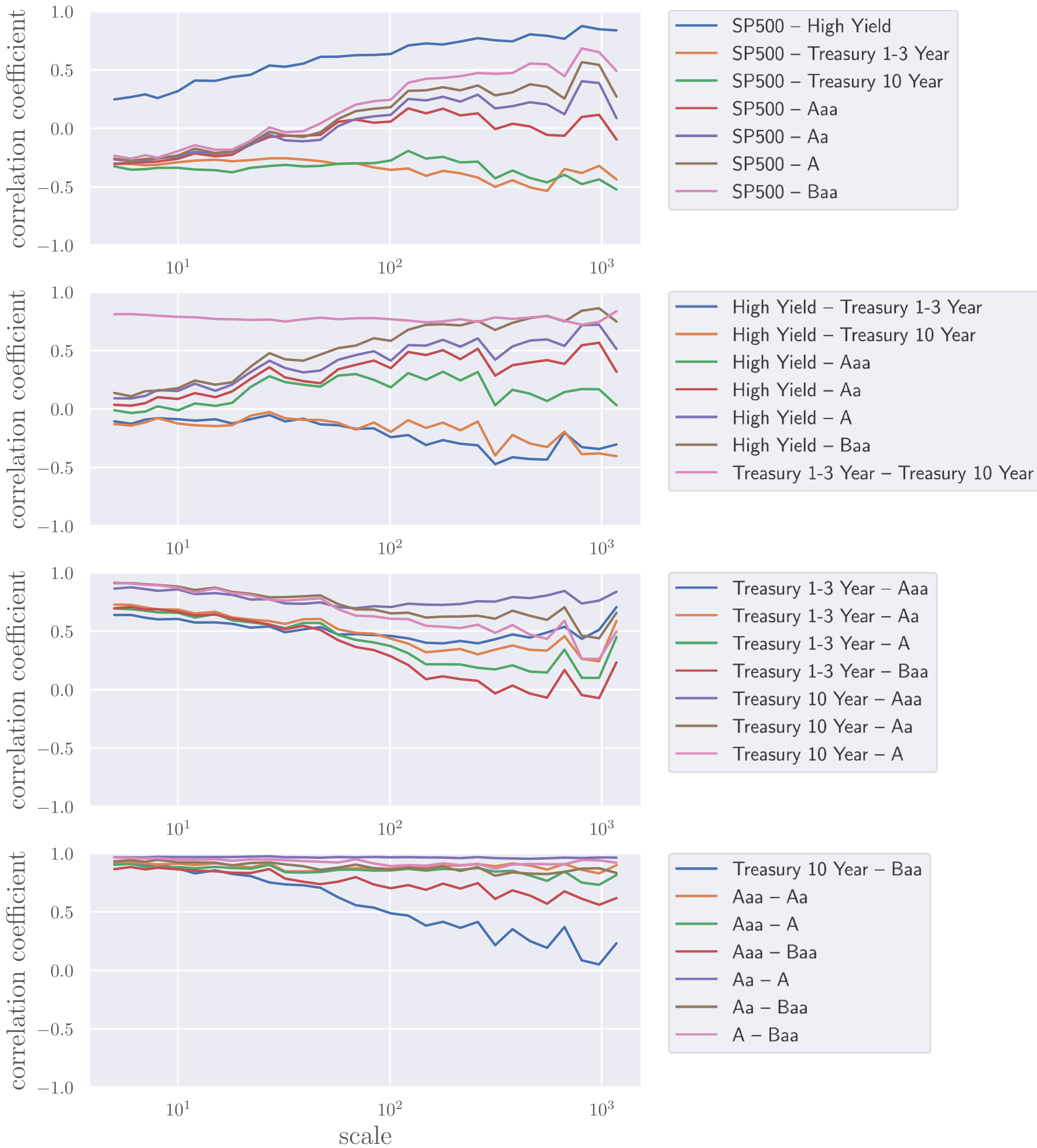


Figure 1. Multi-scale cross-correlation coefficients of the investigated assets.

Table 1. Maximum drawdown performance (%) of the portfolios under a required annual return level of 2.5%.

Sample period	MV	MD-balanced	MD-short	MD-long
2004–2005	−2.227	−2.222	−2.229	**−2.215**
2006–2007	−0.584	−0.533	−0.542	**−0.524**
2008–2009	−1.919	−1.610	**−1.515**	−1.706
2010–2011	−0.662	−0.674	**−0.665**	−0.683
2012–2013	−1.417	−1.206	−1.238	**−1.174**
2014–2015	−1.913	−1.684	**−1.657**	−1.710
2016–2017	−0.606	**−0.518**	−0.540	−0.540
2018–2019	−0.508	−0.480	**−0.469**	−0.492
2020–2021	−2.557	−1.888	−1.990	**−1.786**

The bold values represent the portfolio with the best performance.

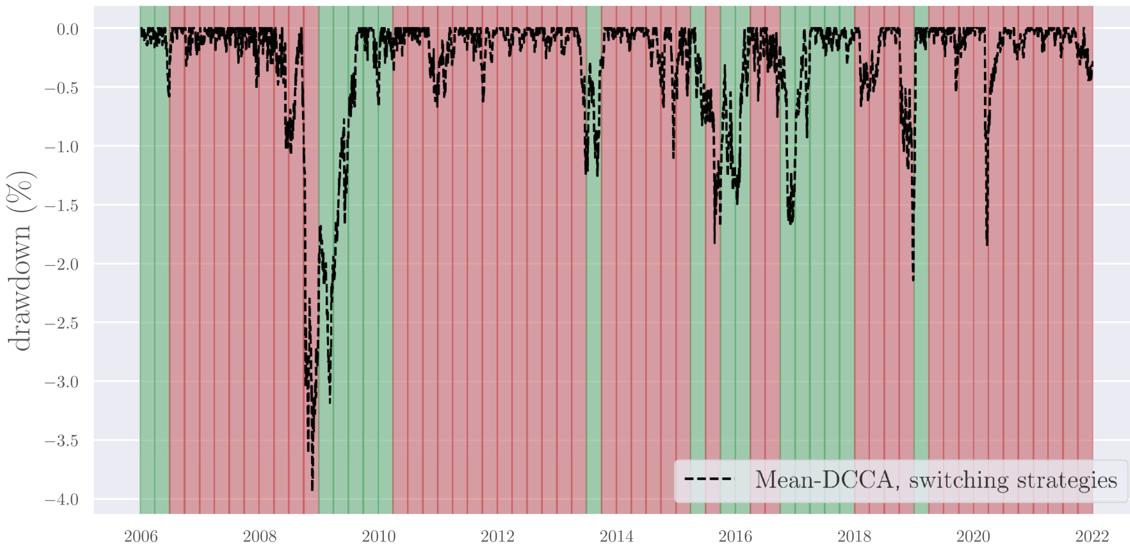
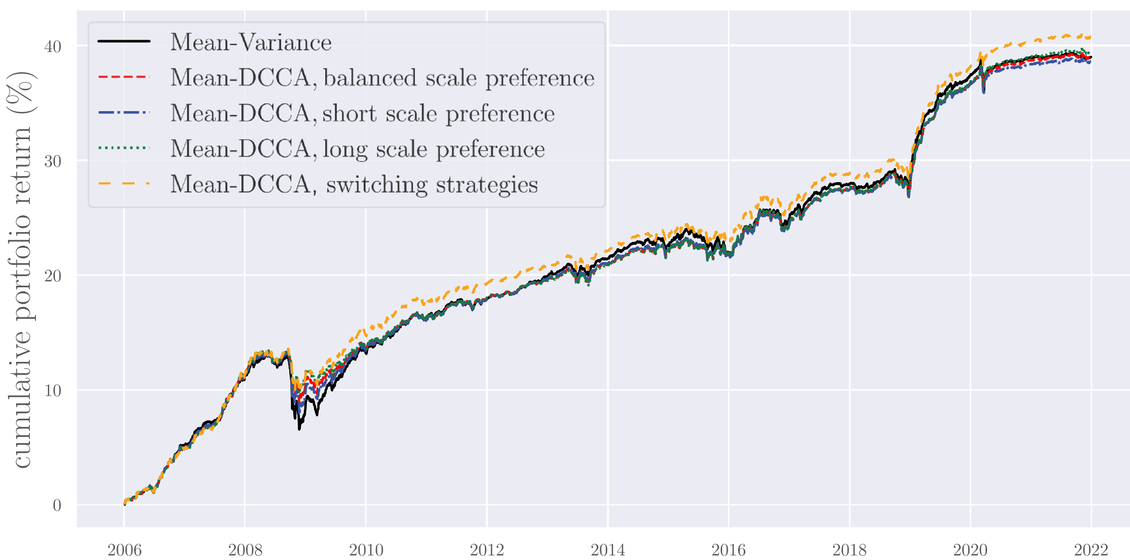
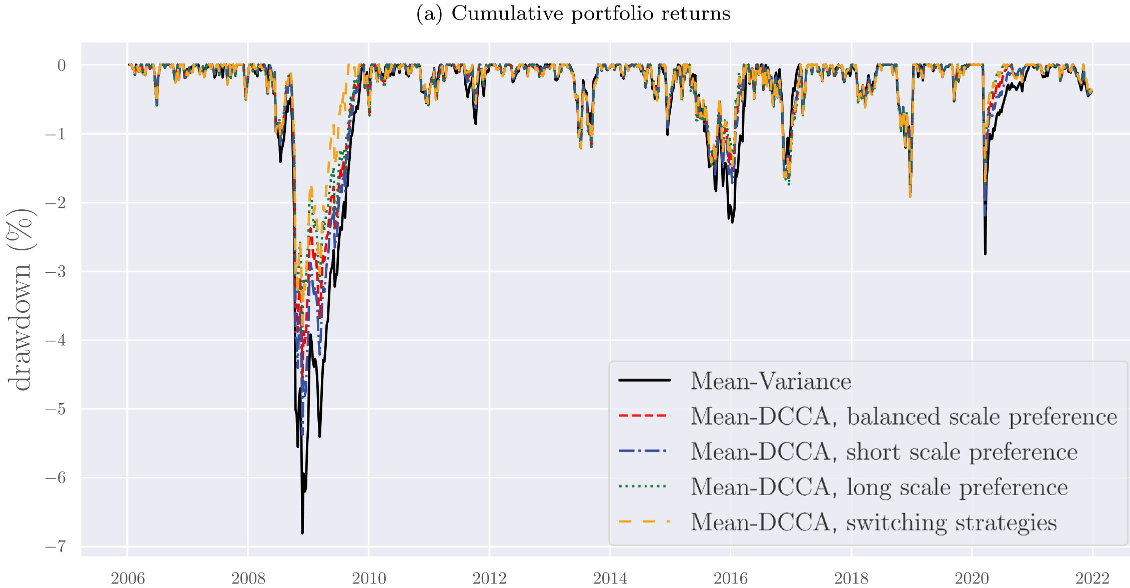


Figure 2. Strategy switching and maximum drawdown throughout the period. The red (green) ranges represent long (short) scale preference.



(a) Cumulative portfolio returns



(b) Maximum drawdown

Figure 3. Comparison of the backtest results for MV and MD portfolio strategies.

Table 2. Backtest performance (annual average) of portfolios.

	MV	MD-balanced	MD-short	MD-long	MD-switching
max. drawdown (%)	−1.537	−1.303	−1.361	−1.255	**−1.240**
10-day 99%VaR (%)	−0.832	−0.725	−0.760	−0.696	**−0.694**
10-day 97.5%ES (%)	−0.809	−0.706	−0.737	−0.678	**−0.675**
120-day 99%VaR (%)	−0.864	−0.591	−0.678	−0.510	**−0.474**
120-day 97.5%ES (%)	−0.832	−0.570	−0.654	−0.490	**−0.455**
annual return (%)	2.440	2.441	2.416	2.466	**2.550**

The bold values represent the portfolio with the best performance.

MD portfolio in response to market conditions is key to allocate more effectively.

In addition to investors' heterogeneity and time-varying dependencies, data frequency matters in various financial concepts (Bannigidadmath and Narayan 2016; Narayan and Sharma 2015; Narayan, Ahmed, and Narayan 2015). Investigating whether the present results are robust to using different frequencies will reveal a further understanding of the profitability of fractal portfolios, which is our future work.

IV. Conclusion

This article applies the fractal MD portfolio to the US market and investigates scale dependence to find opportunities for greater portfolio risk control. Since investors' scale dependence shifts to a different level according to changing economic conditions, further development is achieved by incorporating time-varying features in the MD portfolio. We find empirical evidence that short-scale preference strategy increases diversification effect when the market outlook is uncertain, while long-scale preference strategy is more effective when the market is less turbulent. Such a strategy works well for both in-sample and out-sample data with improvement in the maximum drawdown, VaR, and ES. The results imply that market condition is useful for making a decision at which scale preference the fractal portfolio should be evaluated. In real terms, investors switch their preference oftentimes, potentially providing higher profitability and diversification. An extension of our study would be to examine frequency data dependence on fractal portfolio performance. Another extension would be to consider the multifractality of the assets by introducing a multifractal type of risk function. This may allow for more sophisticated portfolio diversification.

Disclosure statement

The views, thoughts, and opinions expressed in this article belong solely to the authors, and do not reflect the official policy or position of Mizuho Bank or Mizuho Research & Technologies.

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References

Bannigidadmath, D., and P. K. Narayan. 2016. "Stock Return Predictability and Determi- Nants of Predictability and Profi." *Emerging Markets Review* 26:153–173. https://doi.org/10.1016/j.ememar.2015.12.003.

Battiston, S., J. D. Farmer, A. Flache, D. Garlaschelli, A. G. Haldane, H. Heesterbeek, C. Hommes, C. Jaeger, R. May, and M. Scheffer. 2016. "Complexity Theory and Financial Regulation." *Science* 351 (6275): 818–819. https://doi.org/10.1126/science.aad0299.

Evans, M. D. D. 1994. "Expected Returns, Time-Varying Risk, and Risk Premia." *The Journal of Finance* 49 (2): 655–679. https://doi.org/10.1111/j.1540-6261.1994.tb05156.x.

Fama, E. F. 1991. "Efficient Capital Markets: II." *The Journal of Finance* 46 (5): 1575–1617. https://doi.org/10.1111/j.1540-6261.1991.tb04636.x.

Ferson, W. E., and R. W. Schadt. 1996. "Measuring Fund Strategy and Performance in Changing Economic Conditions." *The Journal of Finance* 51 (2): 425–461. https://doi.org/10.1111/j.1540-6261.1996.tb02690.x.

Kristoufek, L. 2018. "Fractality in Market Risk Structure: Dow Jones Industrial Com- Ponents Case." *Chaos, Solitons & Fractals* 110:69–75. https://doi.org/10.1016/j.chaos.2018.02.028.

Markowitz, H. 1952. "Portfolio Selection." *The Journal of Finance* 7 (1): 77. https://doi.org/10.2307/2975974.

Narayan, P. K., H. A. Ahmed, and S. Narayan. 2015. "Do Momentum-Based Trading Strate- Gies Work in the Commodity Futures Markets?" *Journal of Futures Markets* 35 (9): 868–891. https://doi.org/10.1002/fut.21685.

Narayan, P. K., and S. S. Sharma. 2015. "Does Data Frequency Matter for the Impact of Forward Premium on Spot

Exchange Rate?" *International Review of Financial Analysis* 39:45–53. https://doi.org/10.1016/j.irfa.2015.01.011.

Peters, E. E. 1994. *Fractal Market Analysis: Applying Chaos Theory to Investment and Economics*. Vol. 24. New York: John Wiley & Sons.

Podobnik, B., and H. E. Stanley. 2008. "Detrended Cross-Correlation Analysis: A New Method for Analyzing Two Nonstationary Time Series." *Physical Review Letters* 100 (8). https://doi.org/10.1103/PhysRevLett.100.084102.

Tilfani, O., P. Ferreira, and M. Y. E. Boukfaoui. 2019. "Building Multi-Scale Portfolios and Efficient Market Frontiers Using Fractal Regressions." *Physica A: Statistical Mechanics and Its Applications* 532:121758. https://doi.org/10.1016/j.physa.2019.121758.

Tilfani, O., P. Ferreira, and M. Y. E. Boukfaoui. 2020. "Multiscale Optimal Portfolios Using CAPM Fractal Regression: Estimation for Emerging Stock Markets." *Post-Communist Economies* 32 (1): 77–112. https://doi.org/10.1080/14631377.2019.1640983.

Zebende, G. 2011. "DCCA Cross-Correlation Coefficient: Quantifying Level of Cross- Correlation." *Physica A: Statistical Mechanics and Its Applications* 390 (4): 614–618. https://doi.org/10.1016/j.physa.2010.10.022.

Zhang, X., X. Wu, L. Zhang, and Z. Chen. 2022. "The Evaluation of Mean-Detrended Cross-Correlation Analysis Portfolio Strategy for Multiple Risk Assets." *Evaluation Review* 46 (2): 138–164. https://doi.org/10.1177/0193841X221078642.